



LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

UCRL-PROC-309694

Theoretical Challenges of Determining Low-Energy Neutron Capture Cross Sections via the Surrogate Technique

C. Forssén, L. Ahle, L. A. Bernstein, J. A. Church,
F. S. Dietrich, J. Escher, R. D. Hoffman

The Eighth International Symposium on
Nuclei in the Cosmos
Vancouver, BC, Canada
July 19-23, 2004

July 19, 2004

Disclaimer

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

Theoretical challenges of determining low-energy neutron capture cross sections via the Surrogate Technique

C. Forssén^{a*}, L. Ahle^a, L. A. Bernstein^a, J. A. Church^a, F. S. Dietrich^a, J. Escher^a, and R. D. Hoffman^a

^aLawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, CA 94551, United States

Cross sections for radiative neutron capture on unstable nuclei at low energies are difficult to calculate with high precision, and can be impossible to measure directly. It is therefore important to explore alternative methods. The prospects of one such method, the Surrogate Nuclear Reaction Technique, is currently being investigated at Lawrence Livermore National Laboratory. The purpose of this paper is to outline the strategy for combining the results from a surrogate experiment with theoretical calculations in order to extract the desired cross section.

1. INTRODUCTION

The Surrogate Reaction Technique is an innovative method for indirectly determining cross sections for a particular type of reaction, namely two-step reactions proceeding through a compound nucleus (CN). Note that a large number of capture cross sections relevant to astrophysics are of this type, and that they sometimes can be very difficult to study directly due to target or beam limitations. The basic idea of the Surrogate approach is to replace the first step of the “desired” reaction by an alternate (“surrogate”) reaction that populates the same CN. The subsequent decay of the CN into the relevant channel can then be measured and used to extract the desired cross section.

A general overview of the Surrogate project at LLNL, together with the historical background of the method, can be found in Ref. [1]. In the following, the theoretical framework will be outlined, and the main challenges will be highlighted. Particular attention will be devoted to the mismatch between the spin-parity distribution of the intermediate state populated in a low-energy, neutron-capture reaction versus that produced by a surrogate, direct reaction.

2. HAUSER-FESHBACH FORMALISM

The analysis of surrogate reactions relies heavily on CN reaction models. Let us consider the case of a reaction leading from an incoming channel $a + A (= \alpha)$ via an intermediate

*This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory, under contract No. W-7405-Eng-48. The project (04-ERD-057) is funded by the Laboratory Directed Research and Development Program at LLNL.

state B^* to an outgoing channel $c + C (= \chi)$. Using Hauser-Feshbach theory, which rigorously conserves total angular momentum and parity, the energy-averaged cross section $\sigma_{\alpha\chi}(E)$ for this reaction is written as a sum

$$\sigma_{\alpha\chi}(E) = \sum_{J,\Pi} \sigma_{\alpha\chi}(E, J, \Pi), \quad (1)$$

where we use E to denote the excitation energy of the CN. Assuming that the Bohr hypothesis of independence between formation and decay holds separately for each value of total angular momentum and parity, we can write

$$\sigma_{\alpha\chi}(E, J, \Pi) = \sigma_{\alpha}^{CN}(E, J, \Pi) \frac{\Gamma_{\chi}(E, J, \Pi)}{\sum_{\chi'} \Gamma_{\chi'}(E, J, \Pi)}. \quad (2)$$

The CN formation cross section, $\sigma_{\alpha}^{CN}(E, J, \Pi)$, and the decay width, $\Gamma_{\chi}(E, J, \Pi)$, of the CN in channel χ are energy-averaged quantities. The denominator contains a sum over all open channels and thus represents the total decay width. In general there are correlations between the widths which distorts this simple picture. These correlations can be described by width fluctuation factors, $W_{\alpha\chi}$, but we will for simplicity omit them in our formulae.

Hauser-Feshbach theory, which is based on an explicit average over resonances, gives results that can be compared to optical-model calculations, which also yields energy-averaged reaction cross sections. This leads to a relationship between the decay widths and the transmission coefficients describing the absorption in the (complex) optical potential

$$\frac{\Gamma_{\chi}(E, J, \Pi)}{\sum_{\chi'} \Gamma_{\chi'}(E, J, \Pi)} = \frac{\sum_{\{j_{\chi}\}} f(\{j_{\chi}\}, J, \Pi) T_{\{j_{\chi}\}}(\chi)}{\sum_{\chi'} \sum_{\{j_{\chi'}\}} f(\{j_{\chi'}\}, J, \Pi) T_{\{j_{\chi'}\}}(\chi')}, \quad (3)$$

where $\{j_{\chi}\}$ indicates the set of quantum numbers required to specify a particular channel χ , and the factor $f(\{j_{\chi}\}, J, \Pi)$ ensures angular momentum and parity conservation.

In many cases one cannot measure the cross section of a reaction leading to a single final state, but one measures the cross section for populating all the available final states corresponding to a residual nucleus C with energy between U_C and $U_C + dU_C$. Introducing the level density, $\rho(U_C, I_C, \pi_C)$, we arrive at the following Hauser-Feshbach formula

$$\frac{d\sigma_{\alpha\chi}}{dU_C} = \sum_{J,\Pi} \sigma_{\alpha}^{CN}(E, J, \Pi) \frac{\sum_{\{j_{\chi}\}} f(\{j_{\chi}\}, J, \Pi) T_{\{j_{\chi}\}}(\chi) \rho(U_C, I_C, \pi_C)}{\sum_{\chi'} \int_0^{U_C^{\max}} dU_{C'} \sum_{\{j_{\chi'}\}} f(\{j_{\chi'}\}, J, \Pi) T_{\{j_{\chi'}\}}(\chi') \rho(U_{C'}, I_{C'}, \pi_{C'})}. \quad (4)$$

The integral in the Hauser-Feshbach denominator sums over all states in the residual nucleus of channel χ' that are energetically reachable. Note that the level density reduces to a product of delta functions for discrete states. In general the evaluation of the Hauser-Feshbach denominator is plagued by large uncertainties stemming from the level density description. This fact is, e.g., reflected in the very large error bars of many calculated (n, γ) cross sections relevant for astrophysics.

We summarize this section by introducing a generalized Hauser-Feshbach expression written as a sum over products of formation cross sections and branching ratios

$$\sigma_{\alpha\chi}(E) = \sum_{J,\Pi} \sigma_{\alpha}^{CN}(E, J, \Pi) G_{\chi}^{CN}(E, J, \Pi). \quad (5)$$

3. APPLICATION TO SURROGATE REACTIONS

Let us now apply the results of the previous section to the analysis of a surrogate reaction. The goal is to obtain the cross section for a reaction from a channel α to a channel χ , where the target A in channel α is unstable. The Surrogate approach to this problem would be to produce the highly excited compound nucleus B^* in a surrogate reaction $D(d, b)B^*$ with an angle- and energy-differential cross section

$$\frac{d\sigma_{\delta\beta}}{dE d\Omega}(E, \Omega) = \sum_{J, \Pi} \frac{d\sigma_{\delta\beta}}{dE d\Omega}(E, \Omega, J, \Pi), \quad (6)$$

where the left-hand side is the experimentally observable cross section, and the $J\Pi$ on the right-hand side denote different states of the CN populated via the incoming channel δ of the surrogate reaction. This decomposition must be provided by a reaction model calculation. Assuming that the final state of the $D(d, b)B^*$ reaction is a fully equilibrated system we can define the probability that the reaction produces B^* with the quantum numbers $J\Pi$ as

$$F_{\delta\beta}^{CN}(E, \Omega, J, \Pi) = \frac{d\sigma_{\delta\beta}}{dE d\Omega}(E, \Omega, J, \Pi) \bigg/ \sum_{J', \Pi'} \frac{d\sigma_{\delta\beta}}{dE d\Omega}(E, \Omega, J', \Pi'). \quad (7)$$

In a surrogate experiment the ejectile b is measured in coincidence with an observable (e.g., a fission fragment or an emitted γ) that tags the channel χ of the desired reaction. Note that the energy of the ejectile (in combination with the scattering angle), the excitation energy of the CN, and the corresponding energy of the incoming projectile in the desired reaction, are all related to each other. The energy E in our formula can be interpreted as any of these quantities. The experimentally observed probability of this coincidence is

$$P_{\chi}^{CN}(E, \Omega) = \sum_{J', \Pi'} F_{\delta\beta}^{CN}(E, \Omega, J, \Pi) G_{\chi}^{CN}(E, J, \Pi). \quad (8)$$

Let us consider a specific example: The reaction $^{92}\text{Zr}(\alpha, \alpha'\gamma)^{92}\text{Zr}$ was studied in a recent experiment by J. Church *et al.* [2]. This reaction can be used as a surrogate for $^{91}\text{Zr}(n, \gamma)^{92}\text{Zr}$. In the experiment, gammas from the decaying $^{92}\text{Zr}^*$ were observed in coincidence with inelastically scattered α particles. The coincidence probability, $P_{\gamma}^{CN}(E, \theta)$, was measured as a function of ejectile energy and scattering angle. The goal is to compare the final, extracted $\sigma_{n\gamma}$ cross section from the surrogate analysis with existing direct measurements. This experiment will then serve as a benchmark that can help to validate the technique.

3.1. The angular momentum mismatch

Eqs. (5) and (8) constitute the Hauser-Feshbach formulation of the Surrogate reaction method. These expressions contain the same branching ratios, $G_{\chi}^{CN}(E, J, \Pi)$, but they are weighted differently because the $J\Pi$ distributions are different in the two reactions. In fact, this mismatch can be quite significant. The use of the Surrogate technique to extract low-energy (n, γ) cross sections is actually a rather extreme case since the neutron will bring in very little angular momentum to the CN compared with that typically

brought in by the $D(d,b)B^*$ direct reaction. The main theoretical challenge in this case is therefore to determine the spin-parity decomposition probabilities $F_{\delta\beta}^{CN}(E, \Omega, J, \Pi)$ so that the branching ratios $G_{\chi}^{CN}(J, \Pi)$ can be accurately extracted from the observed coincidence probabilities, $P_{\chi}^{CN}(E, \Omega)$, Eq. (8). If this decomposition is well determined, the branching ratios can be inserted in Eq. (5), together with a formation cross section for the desired reaction calculated with an optical potential, to yield the $\sigma_{\alpha\chi}(E)$ cross section. It is particularly desirable for experiments to be carried out over a wide range of angles in the $D(d,b)B^*$ reaction since the probabilities $F_{\delta\beta}^{CN}(E, \Omega, J, \Pi)$ are angle-dependent, and obtaining the same $\sigma_{\alpha\chi}$ for the various angles provides an important consistency check on the procedure.

True independence of formation and decay occurs if the branching ratios of Eq. (5) don't depend on J and Π . This would remove most of the model dependencies from the surrogate analysis. The most important condition that must be fulfilled for this to occur is that the energy of the CN must be significantly high that all channels into which it can decay are dominated by integrals over level densities.

4. SUMMARY

The Surrogate Reaction Technique is a very promising method to extract cross sections that are hard to measure directly. However, as all indirect methods, it requires a combined effort from theorists and experimentalists. The most obvious theoretical challenge is to account for the difference in $J\Pi$ population of the CN. This requires the ability to model the population probabilities entering Eq. (7), which may be a demanding task for direct reaction theory since we are interested in reactions populating unbound states of the intermediate nucleus.

A clear limitation of the method is the fact that it is only able to extract the equilibrium part of the desired reaction. Any pre-equilibrium contributions must be calculated separately, and added to the total cross section. Furthermore, the possibility that the surrogate reaction includes contributions that cannot be satisfactorily described by a purely direct reaction mechanism must also be considered (or experimentally excluded) as this may affect the $J\Pi$ population of the intermediate state.

An example of a very successful application of the Surrogate technique can be found in the work of Younes and Britt [3]. They used (t,p) reactions to populate fissile systems in the actinide region, and used the observed fission probabilities to extract (n,f) cross sections. Their work demonstrates the possibilities and limitations of the method, but also highlights the need for benchmark applications.

REFERENCES

1. J. Escher *et al.*, *The Eighth International Symposium on Nuclei in the Cosmos*, Vancouver, 2004.
2. J. A. Church *et al.*, *The Eighth International Symposium on Nuclei in the Cosmos*, Vancouver, 2004.
3. W. Younes and H. C. Britt, Phys. Rev. C 67 (2003) 024610.